DESIGN OF VARIABLE CONTROL CHARTS UNDER TYPE-2 FUZZY SETS

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ABSTRACT
Control charts are very effective tools to follow process variation and to improve process quality. They can be used to analyze whether the process includes any special causes of variation or not. They can determine assignable causes and give suggestions related with correct actions. So it is completely critical to increase sensitiveness and flexibility of control charts. By the way some data related with control charts can include "uncertainty" or "vagueness" related with process and human evaluations. The fuzzy set theory (FST) is one of the most important tools to solve these problems. We know that the control charts designed with FST are more usable and more preferable for monitoring the process. In this paper, an extension of FST named type-2 fuzzy sets have been used to construct variable control charts (VCCs). For this aim, x-R and x-S control charts have been designed by using typer-2 fuzzy sets. The control limits and center line formulas have been obtained based on type-2 fuzzy sets.

Keywords: Control charts, variable, the fuzzy set theory, type-2 fuzzy sets

1. INTRODUCTION
Control charts (CCs) are very useful tools to monitor process' situation and to warn users about unexpected situations of process. They also continuously check the process to classify it as in control or not. As a results of these advantages, CCs are successfully used in many fields. For this aim, CCs that have different types have been improved for different quality characteristics. We know that Shewhart established control charts for monitoring process' quality characteristics in 1924. These charts consist of three parts: (i) a centerline (CL) that means the average value of the quality characteristic, (ii) upper control limit (UCL) and (iii) lower control limit (LCL) that ensure if the process is in-control, the process' variation will fall between these limits. These type of CCs called as classical control charts (CCCs). The use of CCCs to follow variation of the process is satisfactory if the process data are known exactly. Unfortunately, it is not possible to clearly determine the process data. When human judgments and evaluations has an important role in defining the quality characteristics for a process, the CCCs may not be applicable. The obtained results about process' situation cannot be valid. In this case, the fuzzy set theory (FST) developed by Zadeh can be successfully applied to overcome these problems. The FST is an effective tool for modeling uncertainties that come from human judgments [1]. FST has a great capability to represent vagueness and uncertainties of quality characteristics. By the way, it brings a huge advantage that is related with to obtain more flexibility and more information about process' situation. So the CCs based on FST are more usable when the process data are uncertain or vague; or available information about the process is incomplete or include human evaluations or we desire to collect more information about the process. Although FST has great advantages to model uncertainty, sometimes type-1 fuzzy sets called classical

fuzzy sets cannot model the uncertainty as a result of crisp definitions for membership functions. So, type-2 fuzzy sets that includes fuzzy membership functions proposed by Zadeh in 1975 to improve modeling quality of uncertainty. Membership functions of classical fuzzy sets consist of two-dimensional, whereas membership functions of type-2 fuzzy sets are three-dimensional.This extension provides additional degrees of freedom that directly model uncertainties. Thus type-2 fuzzy sets can successfully represent the uncertainty and can reduce its harmful effects. These originalities can be adopted for CCs. In this paper, the variable control charts (VCCs) have been re-designed with respect to usage of type-2 fuzzy sets that enable to modelling human judgments, vagueness and uncertainties as a result of membership functions of them that are type-1 fuzzy sets and are not crisp numbers. For this aim, two of the most used VCCs named x-R and x-S have been produced under type-2 fuzzy sets. The control limits and center lines have been formulated. The rest of this paper has been organized as follows: A brief introduction related with the fuzzy sets and Type 2 Fuzzy Sets have been summarized into Section 2. A literature review about usage of the fuzzy set theory on control charts has been summarized in Section 3. The VCCs based on type-2 fuzzy sets are detailed in Section 4. The obtained results and future research directions discussed into Section 5.

2. THE FUZZY SETS AND TYPE 2 FUZZY SETS

FST is an important tool to provide measuring uncertainty that are associated with human beings' subjective judgments and evaluations including linguistic terms, satisfaction degree and importance degree. The concept of a linguistic variable is very useful in dealing with situations, which are too complex or not well defined to be reasonably described in conventional quantitative expressions [2]. Membership functions of type-1 fuzzy sets are crisp sets. For this reason, in cases where the meanings of criteria are not clear, the evaluators do not hold the same opinions and the setting of evaluation is noisy, type-1 fuzzy sets cannot offer effective decision support. In such cases, type-2 fuzzy sets whose membership functions are type-1 fuzzy sets too enables convenient modeling of problem. Type-2 fuzzy sets can be regarded as an extension of type-1 fuzzy sets as shown in Figure 1 [2, 3].

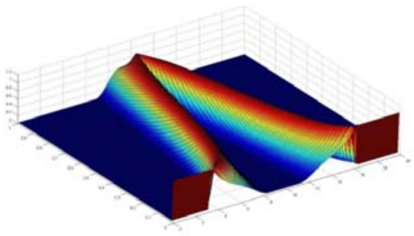

Figure 1 Membership Function of a Type-2 Fuzzy Set

An interval type-2 fuzzy set is a special case of a generalized type-2 fuzzy set. Interval type-2 fuzzy sets are the most commonly used type-2 fuzzy sets because of their simplicity and reduced computational effort with respect to general type-2 fuzzy sets. In this section, the basic concepts and operations of interval type-2 fuzzy sets are introduced below [3, 4, 5]:

A type-2 fuzzy set $\tilde{\tilde{A}}$ in the universe of discourse X can be represented by a type-2 membership function $\mu_{\tilde{\tilde{A}}}$, shown $\tilde{\tilde{A}} = \{((x,u), \mu_{\tilde{\tilde{A}}}(x,u)) | \in X, \forall u \in J_x \subseteq [0,1], 0 \le \mu_{\tilde{\tilde{A}}}(x,u) \le 1\}$ .Where J_x denotes an interval in [0, 1]. If $\mu_{\tilde{\tilde{A}}}(x,u)$ is equal to 1.00 for all values, $\tilde{\tilde{A}}$ named as interval type-2 fuzzy set. Assume that a type-2 fuzzy numbers are defines as $\tilde{\tilde{A}}_i = (\tilde{A}_i^U, \tilde{A}_i^L) = \begin{pmatrix} ((a_{i_1}^U, a_{i_2}^U a_{i_3}^U a_{i_4}^U; H_1(\tilde{A}_i^U), H_2(\tilde{A}_i^U)), \\ (a_{i_1}^L, a_{i_2}^L, a_{i_3}^L, a_{i_4}^L; H_1(\tilde{A}_i^L), H_2(\tilde{A}_i^L)) \end{pmatrix}$

.Where U and L define upper and lower membership values, respectively. The main operations on type-2 fuzzy sets can be summarized as follows:

$$\tilde{\tilde{A}}_1+\tilde{\tilde{A}}_2=\left(\begin{pmatrix}(a^U_{1_1}+a^U_{2_1},a^U_{1_2}+a^U_{2_2},a^U_{1_3}+a^U_{2_3},a^U_{1_4}+a^U_{2_4};\\ \min(H_1(\tilde{A}^U_1),H_1(\tilde{A}^U_2)),\min(H_2(\tilde{A}^U_1),H_2(\tilde{A}^U_2))\end{pmatrix},\begin{pmatrix}(a^L_{1_1}+a^L_{2_1},a^L_{1_2}+a^L_{2_2},a^L_{1_3}+a^L_{2_3},a^L_{1_4}+a^L_{2_4};\\ \min(H_1(\tilde{A}^L_1),H_1(\tilde{A}^L_2)),\min(H_2(\tilde{A}^L_1),H_2(\tilde{A}^L_2))\end{pmatrix}\right)$$

$$\tilde{\tilde{A}}_1/\tilde{\tilde{A}}_2=\left(\begin{pmatrix}(a^U_{1_1}/a^U_{2_4},a^U_{1_2}/a^U_{2_3},a^U_{1_3}/a^U_{2_2},a^U_{1_4}/a^U_{2_1};\\ \min(H_1(\tilde{A}^U_1),H_1(\tilde{A}^U_2)),\min(H_2(\tilde{A}^U_1),H_2(\tilde{A}^U_2))\end{pmatrix},\begin{pmatrix}(a^L_{1_1}/a^L_{2_4},a^L_{1_2}/a^L_{2_3},a^L_{1_3}/a^L_{2_2},a^L_{1_4}/a^L_{2_1};\\ \min(H_1(\tilde{A}^L_1),H_1(\tilde{A}^L_2)),\min(H_2(\tilde{A}^L_1),H_2(\tilde{A}^L_2))\end{pmatrix}\right)$$

$$\tilde{\tilde{A}}_1-\tilde{\tilde{A}}_2=\left(\begin{pmatrix}(a^U_{1_1}-a^U_{2_4},a^U_{1_2}-a^U_{2_3},a^U_{1_3}-a^U_{2_2},a^U_{1_4}-a^U_{2_1};\\ \min(H_1(\tilde{A}^U_1),H_1(\tilde{A}^U_2)),\min(H_2(\tilde{A}^U_1),H_2(\tilde{A}^U_2))\end{pmatrix},\begin{pmatrix}(a^L_{1_1}-a^L_{2_4},a^L_{1_2}-a^L_{2_3},a^L_{1_3}-a^L_{2_2},a^L_{1_4}-a^L_{2_1};\\ \min(H_1(\tilde{A}^L_1),H_1(\tilde{A}^L_2)),\min(H_2(\tilde{A}^L_1),H_2(\tilde{A}^L_2))\end{pmatrix}\right)$$

$$k\otimes\tilde{\tilde{A}}=\left(\begin{array}{l}\left(k\times a^U_{1_1},k\times a^U_{1_2},k\times a^U_{1_3},k\times a^U_{1_4};(H_1(\tilde{A}^U_1),H_1(\tilde{A}^U_2)\right),\\ \left(k\times a^L_{1_1},k\times a^L_{1_2},k\times a^L_{1_3},k\times a^L_{1_4};\left(H_1(\tilde{A}^L_1),H_1(\tilde{A}^L_2)\right)\right)\end{array}\right)$$

$$\tilde{\tilde{A}}_1\otimes\tilde{\tilde{A}}_2=\left(\begin{pmatrix}(a^U_{1_1}\times a^U_{2_1},a^U_{1_2}\times a^U_{2_2},a^U_{1_3}\times a^U_{2_3},a^U_{1_4}\times a^U_{2_4};\\ \min(H_1(\tilde{A}^U_1),H_1(\tilde{A}^U_2)),\min(H_2(\tilde{A}^U_1),H_2(\tilde{A}^U_2))\end{pmatrix},\begin{pmatrix}(a^L_{1_1}\times a^L_{2_1},a^L_{1_2}\times a^L_{2_2},a^L_{1_3}\times a^L_{2_3},a^L_{1_4}\times a^L_{2_4};\\ \min(H_1(\tilde{A}^L_1),H_1(\tilde{A}^L_2)),\min(H_2(\tilde{A}^L_1),H_2(\tilde{A}^L_2))\end{pmatrix}\right)$$

3. LITERATURE REVIEW

The fuzzy set theory has been successfully used into quality control. In this section some papers belongs 2017 to now have been analyzed and classified as shown in Table 1.

Table 1 Usage of the fuzzy sets in control chart

Authors	Type of Fuzzy Sets	Type of Control Charts	Application Area	Authors	Type of Fuzzy Sets	Type of Control Charts	Application Area
Shu et al. [6]	Type-1 (TFN)	x-S	Production	Şentürk and Antuchevicience [11]	Type-2 (TrFN)	*c*	Food Process
Kaya et al. [7]	Type-1 (TFN)	I-MR	Finance	Shabani et al. [12]	Intuitionistic (TrFN)	x-R, x-S	-
Madadi and Mahmoudzadeh [8]	Type-1 (TFN)	*p*	Production	Erginel et al. [13]	Type-2 (TrFN)	*p, np*	Production
Sakthivel et al. [9]	Type-1 (TFN)	*p*	Production	Teksen and Anagün [14]	Type-2 (TrFN)	*c*	-
Şentürk [10]	Type-1 (TFN)	*c*	Production	Fadaei and Pooya [15]	Type-1 (TFN)	*u*	Production

As a result of literature review and analysis, type-2 fuzzy sets can be applied on control charts since they don't have sufficiently take into account in the literature. So, in this paper x-R and x-S control charts have been developed by using type-2 fuzzy sets.

4. VARIABLES CONTROL CHARTS BASED ON TYPE-2 FUZZY SETS

4.1. $\bar{\tilde{\tilde{X}}}$ - $\tilde{\tilde{R}}$ Control Charts

Assume that a type-2 fuzzy number is defined as $\tilde{\tilde{A}}=\left(\tilde{A}^U_i,\tilde{A}^L_i\right)=\left(\begin{array}{l}\left(a^U_{i1},a^U_{i2},a^U_{i3},a^U_{i4};H_1\left(\tilde{A}^U_i\right),H_2\left(\tilde{A}^U_i\right)\right),\\ \left(\left(a^L_{i1},a^L_{i2},a^L_{i3},a^L_{i4};H_1\left(\tilde{A}^L_i\right),H_2\left(\tilde{A}^L_i\right)\right)\right)\end{array}\right)$.

The sampling mean, range and global mean can be computed as shown in below:

$$\bar{\tilde{\tilde{X}}}_{rj}=\frac{\sum X_{rij}}{n},\ \bar{\bar{\tilde{\tilde{X}}}}=\frac{\sum\bar{\tilde{\tilde{X}}}_{rj}}{m} \tag{1}$$

$$\tilde{\tilde{\bar{\bar{X}}}} = \begin{pmatrix} \left(\overline{\overline{X}}_{a_1^U}, \overline{\overline{X}}_{a_2^U}, \overline{\overline{X}}_{a_3^U}, \overline{\overline{X}}_{a_4^U}; \min(H_1(\tilde{\tilde{A}}^U); \min(H_2(\tilde{\tilde{A}}^U)) \right), \\ \left(\overline{\overline{X}}_{a_1^L}, \overline{\overline{X}}_{a_2^L}, \overline{\overline{X}}_{a_3^L}, \overline{\overline{X}}_{a_4^L}; \min(H_1(\tilde{\tilde{A}}^L); \min(H_2(\tilde{\tilde{A}}^L)) \right) \end{pmatrix} \tag{2}$$

$$\tilde{\tilde{R}}_j = \begin{pmatrix} \begin{pmatrix} X_{\max a_{1j}^U} - X_{\max a_{4j}^U}, X_{\max a_{2j}^U} - X_{\max a_{3j}^U}, X_{\max a_{3j}^U} - X_{\max a_{2j}^U}, \\ X_{\max a_{4j}^U} - X_{\max a_{1j}^U}; \min(H_1(\tilde{\tilde{A}}^U), \min(H_2(\tilde{\tilde{A}}^U)) \end{pmatrix}, \\ \begin{pmatrix} X_{\max a_{1j}^L} - X_{\max a_{4j}^L}, X_{\max a_{2j}^L} - X_{\max a_{3j}^L}, X_{\max a_{3j}^L} - X_{\max a_{2j}^L}, \\ X_{\max a_{4j}^L} - X_{\max a_{1j}^L}; \min(H_1(\tilde{\tilde{A}}^L), \min(H_2(\tilde{\tilde{A}}^L)) \end{pmatrix} \end{pmatrix} \tag{3}$$

Where $r = a_1^U, a_2^U, a_3^U, a_4^U, a_1^L, a_2^L, a_3^L, a_4^L$ and $i = 1,2,3,4,....,n$ and $j = 1,2,3,4,...,m$.

$$\tilde{\tilde{R}}_j = \begin{pmatrix} \left(R_{a_{1j}^U}, R_{a_{2j}^U}, R_{a_{3j}^U}, R_{a_{4j}^U}; \min(H_1(\tilde{\tilde{A}}^U); \min(H_2(\tilde{\tilde{A}}^U) \right), \\ \left(R_{a_{1j}^L}, R_{a_{2j}^L}, R_{a_{3j}^L}, R_{a_{4j}^L}; \min(H_1(\tilde{\tilde{A}}^L); \min(H_2(\tilde{\tilde{A}}^L) \right) \end{pmatrix} \tag{4}$$

$$\tilde{\tilde{\bar{R}}} = \begin{pmatrix} \left(\frac{\sum R_{a_{1j}^U}}{m}, \frac{\sum R_{a_{2j}^U}}{m}, \frac{\sum R_{a_{3j}^U}}{m}, \frac{\sum R_{a_{4j}^U}}{m}; \min(H_1(\tilde{\tilde{A}}^U); \min(H_2(\tilde{\tilde{A}}^U) \right), \\ \left(\frac{\sum R_{a_{1j}^L}}{m}, \frac{\sum R_{a_{2j}^L}}{m}, \frac{\sum R_{a_{3j}^L}}{m}, \frac{\sum R_{a_{4j}^L}}{m}; \min(H_1(\tilde{\tilde{A}}^L); \min(H_2(\tilde{\tilde{A}}^L) \right) \end{pmatrix} \tag{5}$$

Then the control limits for $\tilde{\tilde{\bar{\bar{X}}}}$ control charts can be computed as follows:

$$\tilde{\tilde{UCL}}_{\bar{x}} = \begin{pmatrix} \begin{pmatrix} \overline{\overline{X}}_{a_1^U} + A_2 \overline{R}_{a_1^U}, \overline{\overline{X}}_{a_2^U} + A_2 \overline{R}_{a_2^U}, \overline{\overline{X}}_{a_3^U} + A_2 \overline{R}_{a_3^U}, \overline{\overline{X}}_{a_4^U} + A_2 \overline{R}_{a_4^U}; \\ \min(H_1(\tilde{\tilde{A}}^U); \min(H_2(\tilde{\tilde{A}}^U) \end{pmatrix}, \\ \begin{pmatrix} \overline{\overline{X}}_{a_1^L} + A_2 \overline{R}_{a_1^L}, \overline{\overline{X}}_{a_2^L} + A_2 \overline{R}_{a_2^L}, \overline{\overline{X}}_{a_3^L} + A_2 \overline{R}_{a_3^L}, \overline{\overline{X}}_{a_4^L} + A_2 \overline{R}_{a_4^L}; \\ \min(H_1(\tilde{\tilde{A}}^L); \min(H_2(\tilde{\tilde{A}}^L) \end{pmatrix} \end{pmatrix} \tag{6}$$

$$\tilde{\tilde{CL}}_{\bar{X}} = \begin{pmatrix} \left(\overline{\overline{X}}{}_{a_1}^U, \overline{\overline{X}}{}_{a_2}^U, \overline{\overline{X}}{}_{a_3}^U, \overline{\overline{X}}{}_{a_4}^U; \min(H_1(\tilde{\tilde{A}}^U); \min(H_2(\tilde{\tilde{A}}^U)) \right), \\ \left(\overline{\overline{X}}{}_{a_1}^L, \overline{\overline{X}}{}_{a_2}^L, \overline{\overline{X}}{}_{a_3}^L, \overline{\overline{X}}{}_{a_{i4}}^L; \min(H_1(\tilde{\tilde{A}}^L); \min(H_2(\tilde{\tilde{A}}^L)) \right) \end{pmatrix} \tag{7}$$

$$\tilde{\tilde{LCL}}_{\bar{x}} = \begin{pmatrix} \begin{pmatrix} \overline{\overline{X}}_{a_1^U} - A_2 \overline{R}_{a_4^U}, \overline{\overline{X}}_{a_2^U} - A_2 \overline{R}_{a_3^U}, \overline{\overline{X}}_{a_3^U} - A_2 \overline{R}_{a_2^U}, \overline{\overline{X}}_{a_4^U} - A_2 \overline{R}_{a_1^U}; \\ \min(H_1(\tilde{\tilde{A}}^U); \min(H_2(\tilde{\tilde{A}}^U) \end{pmatrix}, \\ \begin{pmatrix} \overline{\overline{X}}_{a_1^L} - A_2 \overline{R}_{a_4^L}, \overline{\overline{X}}_{a_2^L} - A_2 \overline{R}_{a_3^L}, \overline{\overline{X}}_{a_3^L} - A_2 \overline{R}_{a_2^L}, \overline{\overline{X}}_{a_4^L} - A_2 \overline{R}_{a_1^L}; \\ \min(H_1(\tilde{\tilde{A}}^L); \min(H_2(\tilde{\tilde{A}}^L) \end{pmatrix} \end{pmatrix} \tag{8}$$

By the way, the control limits for $\tilde{\tilde{R}}$ control charts can be computed as follows:

$$\tilde{\tilde{UCL}}_R = \begin{pmatrix} \left(D_4 \overline{R}_{a_1^U}, D_4 \overline{R}_{a_2^U}, D_4 \overline{R}_{a_3^U}, D_4 \overline{R}_{a_4^U}; \min(H_1(\tilde{\tilde{A}}^U)), \min(H_2(\tilde{\tilde{A}}^U)) \right), \\ \left(D_4 \overline{R}_{a_1^L}, D_4 \overline{R}_{a_2^L}, D_4 \overline{R}_{a_3^L}, D_4 \overline{R}_{a_4^L}; \min(H_1(\tilde{\tilde{A}}^L)), \min(H_2(\tilde{\tilde{A}}^L)) \right) \end{pmatrix} \tag{9}$$

$$\tilde{\tilde{CL}}_R = \begin{pmatrix} \left(\overline{R}_{a_1^U}, \overline{R}_{a_2^U}, \overline{R}_{a_3^U}, \overline{R}_{a_4^U}; \min(H_1(\tilde{\tilde{A}}^U)), \min(H_2(\tilde{\tilde{A}}^U)) \right), \\ \left(\overline{R}_{a_1^L}, \overline{R}_{a_2^L}, \overline{R}_{a_3^L}, \overline{R}_{a_4^L}; \min(H_1(\tilde{\tilde{A}}^L)), \min(H_2(\tilde{\tilde{A}}^L)) \right) \end{pmatrix} \tag{10}$$

$$\tilde{\tilde{LCL}}_R = \begin{pmatrix} \left(D_3 \overline{R}_{a_1^U}, D_3 \overline{R}_{a_2^U}, D_3 \overline{R}_{a_3^U}, D_3 \overline{R}_{a_4^U}; \min(H_1(\tilde{\tilde{A}}^U)); \min(H_2(\tilde{\tilde{A}}^U)) \right), \\ \left(D_3 \overline{R}_{a_1^L}, D_3 \overline{R}_{a_2^L}, D_3 \overline{R}_{a_3^L}, D_3 \overline{R}_{a_4^L}; \min(H_1(\tilde{\tilde{A}}^L)); \min(H_2(\tilde{\tilde{A}}^L)) \right) \end{pmatrix} \tag{11}$$

These fuzzy control limits and structure of control chart based on type-2 fuzzy sets have been drawn in Figure 2.

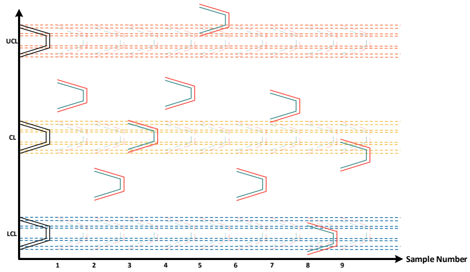

Figure 2 A control chart based on type-2 fuzzy sets

4.2. $\tilde{\tilde{\bar{X}}} - \tilde{\tilde{S}}$ Control Charts

If we use type-2 fuzzy numbers, standard deviation of a sample and mean for these deviations can be computed as shown in below:

$$\tilde{\tilde{s}}_{rj} = \sqrt{\left(\frac{\sum\left(X_{rij} - \bar{X}_{rj}\right)^2}{n-1}\right)}, \qquad \bar{\tilde{\tilde{s}}} = \frac{\sum \tilde{\tilde{s}}_{rj}}{m} \tag{12}$$

Then the membership function of $\bar{\tilde{\tilde{s}}}$ can be obtained as shown in below:

$$\bar{\tilde{\tilde{s}}} = \left(\begin{array}{l}\left(\frac{\sum s_{a_{1j}^U}}{m}, \frac{\sum s_{a_{2j}^U}}{m}, \frac{\sum s_{a_{3j}^U}}{m}, \frac{\sum s_{a_{4j}^U}}{m}; \min(H_1(\tilde{\tilde{A}}^U)); \min(H_2(\tilde{\tilde{A}}^U))\right), \\ \left(\frac{\sum s_{a_{1j}^L}}{m}, \frac{\sum s_{a_{2j}^L}}{m}, \frac{\sum s_{a_{3j}^L}}{m}, \frac{\sum s_{a_{4j}^L}}{m}; \min(H_1(\tilde{\tilde{A}}^L)); \min(H_2(\tilde{\tilde{A}}^L))\right)\end{array}\right) \tag{13}$$

Then the control limits for $\tilde{\tilde{\bar{X}}}$ control chart can be computed as shown in below:

$$\tilde{\tilde{UCL}}_{\bar{x}} = \left(\begin{array}{l}\left(\begin{array}{l}\bar{\bar{X}}_{a_1^U} + A_3 \bar{s}_{a_1^U}, \bar{\bar{X}}_{a_2^U} + A_3 \bar{s}_{a_2^U}, \bar{\bar{X}}_{a_3^U} + A_3 \bar{s}_{a_3^U}, \bar{\bar{X}}_{a_4^U} + A_3 \bar{s}_{a_4^U}; \\ \min(H_1(\tilde{\tilde{A}}^U); \min(H_2(\tilde{\tilde{A}}^U)\end{array}\right), \\ \left(\begin{array}{l}\bar{\bar{X}}_{a_1^L} + A_3 \bar{s}_{a_1^L}, \bar{\bar{X}}_{a_2^L} + A_3 \bar{s}_{a_2^L}, \bar{\bar{X}}_{a_3^L} + A_3 \bar{s}_{a_3^L}, \bar{\bar{X}}_{a_4^L} + A_3 \bar{s}_{a_4^L}; \\ \min(H_1(\tilde{\tilde{A}}^L); \min(H_2(\tilde{\tilde{A}}^L)\end{array}\right)\end{array}\right) \tag{14}$$

$$\tilde{\tilde{CL}}_{\bar{x}} = \left(\begin{array}{l}\left(\bar{\bar{X}}_{a_1^U}, \bar{\bar{X}}_{a_2^U}, \bar{\bar{X}}_{a_3^U}, \bar{\bar{X}}_{a_4^U}; H_1(\tilde{\tilde{A}}^U); H_2(\tilde{\tilde{A}}^U)\right), \\ \left(\bar{\bar{X}}_{a_1^L}, \bar{\bar{X}}_{a_2^L}, \bar{\bar{X}}_{a_3^L}, \bar{\bar{X}}_{a_4^L}; H_1(\tilde{\tilde{A}}^L); H_2(\tilde{\tilde{A}}^L)\right)\end{array}\right) \tag{15}$$

$$\tilde{\tilde{LCL}}_{\bar{x}} = \left(\begin{array}{l}\left(\begin{array}{l}\bar{\bar{X}}_{a_1^U} - A_3 \bar{s}_{a_4^U}, \bar{\bar{X}}_{a_2^U} - A_3 \bar{s}_{a_3^U}, \bar{\bar{X}}_{a_3^U} - A_3 \bar{s}_{a_2^U}, \bar{\bar{X}}_{a_4^U} - A_3 \bar{s}_{a_1^U}; \\ \min(H_1(\tilde{\tilde{A}}^U); \min(H_2(\tilde{\tilde{A}}^U)\end{array}\right), \\ \left(\begin{array}{l}\bar{\bar{X}}_{a_1^L} - A_3 \bar{s}_{a_4^L}, \bar{\bar{X}}_{a_2^L} - A_3 \bar{s}_{a_3^L}, \bar{\bar{X}}_{a_3^L} - A_3 \bar{s}_{a_2^L}, \bar{\bar{X}}_{a_4^L} - A_3 \bar{s}_{a_1^L}; \\ \min(H_1(\tilde{\tilde{A}}^L); \min(H_2(\tilde{\tilde{A}}^L)\end{array}\right)\end{array}\right) \tag{16}$$

By the way, the control limits for $\tilde{\tilde{S}}$ control charts can be obtained as follows:

$$\tilde{\tilde{UCL}}_{s} = \left(\begin{array}{l}\left(B_4 \bar{s}_{a_1^U}, B_4 \bar{s}_{a_2^U}, B_4 \bar{s}_{a_3^U}, B_4 \bar{s}_{a_4^U}; \min(H_1(\tilde{\tilde{A}}^U)); \min(H_2(\tilde{\tilde{A}}^U))\right), \\ \left(B_4 \bar{s}_{a_1^L}, B_4 \bar{s}_{a_2^L}, B_4 \bar{s}_{a_3^L}, B_4 \bar{s}_{a_4^L}; \min(H_1(\tilde{\tilde{A}}^L)); \min(H_2(\tilde{\tilde{A}}^L))\right)\end{array}\right) \tag{17}$$

$$\tilde{\tilde{CL}}_{s} = \left(\begin{array}{l}\left(\bar{s}_{a_1^U}, \bar{s}_{a_2^U}, \bar{s}_{a_3^U}, \bar{s}_{a_4^U}; \min(H_1(\tilde{\tilde{A}}^U)); \min(H_2(\tilde{\tilde{A}}^U))\right), \\ \left(\bar{s}_{a_1^L}, \bar{s}_{a_2^L}, \bar{s}_{a_3^L}, \bar{s}_{a_4^L}; \min(H_1(\tilde{\tilde{A}}^L)); \min(H_2(\tilde{\tilde{A}}^L))\right)\end{array}\right) \tag{18}$$

$$\tilde{LCL}_s = \begin{pmatrix} \left(B_3\overline{s_{a_1^U}}, B_3\overline{s_{a_2^U}}, B_3\overline{s_{a_3^U}}, B_3\overline{s_{a_4^U}}; \min(H_1(\tilde{A}^U)); \min(H_2(\tilde{A}^U))\right), \\ \left(B_3\overline{s_{a_1^L}}, B_3\overline{s_{a_2^L}}, B_3\overline{s_{a_3^L}}, B_3\overline{s_{a_4^L}}; \min(H_1(\tilde{A}^L)); \min(H_2(\tilde{A}^L))\right) \end{pmatrix} \qquad (19)$$

5. CONCLUSIONS

FST can be successfully adopted for CCs to define uncertainty and human judgements. For this aim some studies based on usage of FST in CCs have been well done recently. But most of them have been designed with respect to type-1 fuzzy sets. We know that type-2 fuzzy sets are more effective than type-1 fuzzy sets to design of CCs. They can successfully reflects uncertainty and vagueness in CCs. In this paper, well-known two of VCCs named x-R and x-S have been designed by using type-2 fuzzy sets. For this aim, UCL, CL and LCL values of these charts have been formulated. As future research directions, the proposed new CCs have been used on a real case application and the obtained results can be compared with traditional CCs. By the way, the other types of CCs such as *p, np, c* and *u* can be designed with respect to type-2 fuzzy sets.

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