ENTRY FEES AND BIDDER BEHAVIOR IN SEALED - BID AUCTIONS

TROŠKOVI ULASKA I PONAŠANJE PONUĐAČA U AUKCIJAMA SA ZATVORENIM PONUDAMA

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ABSTRACT
Auctions are vital in economic transactions, from private sales to government procurement. A key aspect is bidders' participation decisions, especially when entry incurs costs. This paper compares two models: Levin and Smith (1994), where bidders pay entry fees before learning valuations—concluding fees are never optimal—and Menezes and Monteiro (2000), where bidders first learn valuations, showing fees can screen low-value participants and increase revenue. The paper explores theoretical foundations, equilibrium strategies, and revenue implications, offering insights into endogenous entry, revenue equivalence, and bidder behavior in sealed-bid auctions. This research enhances auction design understanding in settings with costly participation.

Keywords: auctions, bidder participation, entry costs

REZIME
Aukcije igraju ključnu ulogu u ekonomskim transakcijama, od privatnih prodaja do javnih nabavki. Ključni aspekt je odluka ponuđača o učešću, posebno kada ulazak nosi troškove. Ovaj rad upoređuje dva modela: Levin i Smith (1994), gdje ponuđači plaćaju naknade za ulazak prije nego što saznaju svoje procjene vrijednosti—zaključujući da naknade nikada nisu optimalne—i Menezes i Monteiro (2000), gdje ponuđači prvo saznaju svoje procjene vrijednosti, pokazujući da naknade mogu filtrirati učesnike s niskim vrijednostima i povećati prihod. Rad istražuje teorijske osnove, ekvilibrijum strategije i implikacije na prihode, pružajući uvid u endogeni ulazak, ekvivalenciju prihoda i ponašanje ponuđača u aukcijama sa zatvorenim ponudama. Ovo istraživanje omogućava dublje razumijevanje dizajna aukcija u situacijama gdje učešće nosi određene troškove.

Ključne riječi: aukcije, učešće ponuđača, troškovi ulaska

1. INTRODUCTION
Auctions have been practiced since ancient times, with historical evidence suggesting that even the Roman Empire was once sold through an auction to the next emperor (Engelbrecht-Wiggans, Shubik, and Stark 1983). They continue to play a crucial role both practically and theoretically, as a significant portion of economic transactions occurs through them. Over time, various auction formats have emerged, with online auctions becoming the most prominent.
Due to their structured and well-defined nature, auctions serve as an ideal testing ground for economic theories, including game theory under incomplete information. They have contributed to the development of theoretical concepts such as price setting and negotiation processes. Additionally, auctions play a role in diverse areas such as lobbying contests, wars of attrition, rationing, and

competitive tournaments (Klemperer 1999). A key aspect of auction theory is the bidder's decision to participate. Sellers may impose entry fees to filter out low-valuation bidders, while bidders also face inherent costs unrelated to the seller, such as the time and effort required to understand the item, attend the auction, and prepare a bid. Consequently, a substantial body of research focuses on the bidder's entry decision, as it serves as the initial stage that influences the subsequent dynamics of the auction.

Various auction models exist, differing in the timing of cost introduction, the number of bidders, and the broader economic environment. One of the most often cited papers in the auction theory is *"Equilibrium in Auctions with Entry"* by Levin and Smith, published in 1994. The paper introduced a new model of entry that was different from the existing literature in multiple ways. Levin and Smith (1994) introduced mixed entry strategies, showing that the celebrated revenue equivalence theorem still holds in auctions with entry. They also showed that imposing entry fees discourages participation and should never be optimal, similar to reservation prices.

Six years later, in 2000, Menezes and Monteiro published *Auctions with Endogenous Participation*. Just like Levin and Smith, their starting point is the Independent Value Theorem (IPV). Both papers examine the assumption of the IPV model that bidders know the number of competitors they face. Recognizing that this assumption is unrealistic—particularly in sealed-bid auctions where bidders are aware of the potential participant pool but not the exact number of actual bidders—both papers introduce endogenous entry. Additionally, they also introduce participation fees, but only after the bidder sees his valuation. This is not the case in Levin and Smith, where bidders first incur the cost, and then later realize their valuations.

Although these two papers share a common starting point, they differ in how they introduce entry fees, they differ in the introduction of entry fees, which in turn has a direct effect on the further development of the model. This paper aims to analyze the effects of these differences and explore other similarities and distinctions between the two models. Additionally, the paper provides a comprehensive and detailed analysis of the assumptions and methodologies used in auction theory, emphasizing how variations in these factors can lead to different outcomes. It also highlights the limitations of the IPV model assumption and contributes to a more accurate representation of bidder behavior.

2. LITERATURE REVIEW

Auctions have become a popular tool for allocating public resources, such as water rights, power supply contracts, etc. Their most cited advantage is their tendency to attain allocative efficiency without requesting the government (or some other seller) to gain prior knowledge of resource values or costs. Thus, auctions are often used as a policy tool, and their effective use is based on the notion that market mechanisms can be designed to achieve particular outcomes of interest.

Auction is a market process where the comparison of bids determines buyers and sellers, along with the terms of trade, including price (Milgrom 1996). The first major paper that recognized the game-theoretic aspect of the auctions is Vickery (1961). He introduced the second-price sealed-bid auction where each buyer draws a valuation known only to her, from a distribution function known to all buyers in the auction. The rules of the auction specify that all buyers submit the sealed-bid at the same time, where each one has a dominant strategy to submit a bid equal to the value one has drawn.

Dominant strategy maximizes one's expected utility no matter what strategies other players follow. Thus, bidders cannot do better than their true value (Noussair 1993), which is why he showed that they are perfectly economically efficient, and have the same expected revenue for bidders as equilibrium strategies in first-price sealed-bid auctions. However, they seldom appear in practice, as first-price sealed-bids are more popular.

Entry fees are commonly used as instruments that improve the revenue performance of auctions. There are many sources for entry fees. For example, bidders can have transportation costs to arrive at an auction, or to learn the rules of the auction and how to submit a bid, or simply have an opportunity cost of participating in an auction. When these fees are introduced, the behavior of bidders can change.

If the bidder's expected revenue from the auction is less than the participation cost, he will choose not to enter the auction. In the case the bidder does participate, his behavior might be different from the one in the standard auction as the number of bidders is endogenous (Cao and Tian 2010).

Levin and Smith (1994) conclude that positive entry fees maximize the expected revenue. Janssen, Karamychev, and Maasland (2011) give an example of a two-step auction game: first, bidders choose an individual entry fee that is publicly announced, and second, each bidder participates in the auction. As a result of signaling, efficiency is restored, despite the negative externalities. If there were no signaling, the reserve price and entry fee would be the same instruments, as both are used to design an optimal auction (Bos and Truyts 2023). Green and Laffont (1984) analyze second-price auctions with participation cost in a general framework where buyers' valuations and entry fees are private information. They demonstrate the symmetric equilibrium with uniform distributions.

Many other authors study equilibrium in second-price auctions where participation costs are equal, and buyers' values are private information (for example, Miralles (2008), Campbell (1998), Tan and Yilankaya (2006)). Significantly less attention is given to first-price auctions with entry fees, although they are used more in practice, such as public procurement. The reason behind this is that the bidding strategies in first-price auctions are not as explicit since bidders do not bid their true valuation. The extent of shading depends greatly on who else enters the auction, and what information is acquired from the entrance of those bidders. As bidders use different thresholds to enter the auction, there may not be an explicit bidding function, resulting in bidders using different strategies (Cao and Tian, 2010).
Chan, Laplagne, and Appels (2003) analyze the reserve prices in second-price sealed-bid auctions with affiliated values. Moreover, Tsuchihashi (2020) uses the reserve price in first-price sealed-bid auctions with independent private values. Both papers deal with reserve price as a signaling device used by sellers to disclose some additional information to the bidders (Bos and Truyts 2023).

3. THE MODELS

The Independent Private Values (IPV) auction model assumes that the number of potential buyers is commonly known (Matthews 1995). Thus, most of the auction literature assumes that the number of bidders is given. Endogenizing the entry would be a natural step that is necessary to complete the understanding of how auction design affects its performance (Levin and Smith, 1994).

Menezes and Monteiro (2000) consider a sealed-bid auction with endogenous participation, with the sale of one indivisible good. They assume the reserve price to be 0, and note that I is a finite set of potential risk-neutral bidders. Just as in the IPV model, bidder $i \in I$ knows her own value (v_i) for the object. However, bidder i knows only the distribution of $F(v_j)$, $\forall j \neq i$, of other bidders' values. Additionally, they assume that values are independently drawn from the continuous distribution function F with support on $[0, \overline{v}]$. Each bidder incurs participation cost $c \in [0, \overline{v}]$, and has a cutoff point v_ρ. Thus, bidder j will participate if and only if $v_j > v_\rho$. If bidder i bids x, and bidder j bids $b_j = b(v_j), j \neq i$, then i wins if and only if $b^{-1}(x) > v_j$. Then, i's expected utility (in first-price auctions) is:

$$g(x) = g_{v_i}(x) := \pi_i(v_i, x, (b_j)_{j \neq i}) = (v_i - x)\,[F(\max\{b^{-1}(x), v_\rho\})]^{n-1} - c. \qquad (1)$$

When $b^{-1}(x) > v_\rho$, then:

$$g(x) = (v_i - x)\,[F(b^{-1}(x))]^{n-1} - c, \qquad (2)$$

and when $b^{-1}(x) < v_\rho$, then $g(x) \leq g(0)$. Maximizing equation (2) with respect to g reveals that if a bidder has a cutoff value equal to v_ρ, then he is indifferent between entering and not entering the auction. This later implies that the bidder with value v_ρ can win only if he is the only participant in the auction. Menezes and Monteiro (2000) compute the revenue for the first-price auctions as follows:

$$R^1 = E\,[b^*(\max\{v_1, \ldots, v_k\})\,\lambda_{\max\{v_1, \ldots, v_k\} \geq v_\rho}] = \int_{v_\rho}^{\overline{v}} b^*(x) n F^{n-1}(x) f(x)\, dx.$$

When it comes to second-price auctions, the bidder bids his true value, and his profit in equilibrium is function of his own value and the value of the bidders in the auction ($v_{\rho s}$ is now a cutoff point for the second-price auction):

$$\pi_i(v_i, v_{j,\, j\neq i}) = E\,[(v_i - \max_{j\neq i} v_j)^+ \lambda_{\max_{j\neq i}, v_j \geq v_{\rho s}} + v_i\ \lambda_{\max_{j\neq i}, v_j < v_{\rho s}}] - c \quad (4)$$

In such instance, the bidder's expected revenue is the expected value of the second highest bid:

$$R^2 = n(n - 1) \int_{v\rho} \overline{v}(1 - F(x))\, x\, (F(x))^{n-2} f(x)\, dx. \quad (5)$$

Menezes and Monteiro (2000) conclude and prove that in an IPV model with a fixed number of endogenous bidder, symmetric equilibrium in first-price and second-price auctions generate the same expected revenue.

Similar to Menezes and Monteiro (2000), Levin and Smith (1994) deal with auctions with endogenous entry with entry costs. However, their setting is different to a certain extent. Their auction is set in two stages: the first stage is when each participant faces a cost of entry, *c*, and he decides whether to enter the auction or not. Thus, contrary to Menezes and Monteiro (2000), their fee does not screen participants with low valuation. The second stage is when bidding takes place. Just like Menezes and Monteiro (2000), they also assume that the seller and all potential bidders are risk-neutral. Another assumption is that information is symmetric, meaning that all participants randomly draw values from the same distribution. There are other papers that deal with asymmetric information, such as Tan and Yilankaya (2005), Cao and Tian (2010). Levin and Smith (1994) denote the potential bidder's expected gain from entering as $E(\pi \mid n, m)$.
Moreover, every participant has to pay entry cost - *c*, learn the number of bidders - *n*, and build the symmetric Nash strategy implied by *n* and auction mechanism - *m*. They focus on cases where entry costs are low enough to admit some, but not all participants. Thus, there is room only for $n < N$ bidders with mixed strategies, where each potential bidder enters with probability *q*, and does not enter with probability $1 - q$, making *n* varying stochastically between 0 and *N*. In order for some $q^* \in (0, 1)$ to represent equilibrium with mixed strategies, each potential participant has to be indifferent between entering and not entering, i.e.:

$$\sum_{n=1}^{N}[\tbinom{N-1}{n-1}\ (q*)n - 1\ (1 - q^*)^{N-n} E[\pi \mid n, m]] = 0 \quad (6)$$

The first term in equation (6) denotes the binomial probability that exactly $n-1$ other participants enter, given *n* bidders in total. This q^* from (6) characterizes equilibrium in mixed strategies. The total expected profit of all N participants is:

$$B(q, \Omega) = N\, q\, B_i(q, \Omega) = [\sum_{n=1}^{N} p_n T_n(R_n) V_n - \underline{n}c] - [\sum_{n=1}^{N} p_n T_n(R_n) W_n + \overline{n}e], \quad (7)$$

where p_n stands for binomial probability that exactly *n* bidders enter, $T_n(R_n)$ stands for probability of trade given *n* and seller's mechanism, V_n for the expected value of an item to the highest bidder, W_n for expected payment to the seller, $\overline{n}$ for the mean of binomial distribution, and finally *e* for the entry fee. The seller's expected revenue is:

$$\pi(q, \Omega) = \sum_{n=1}^{N} p_n T_n(R_n) W_n + \overline{n}e, \quad (8)$$

and social welfare is:

$$S(q, \Omega) = B(q, \Omega) + \pi(q, \Omega) = \sum_{n=1}^{N} p_n T_n(R_n) V_n - \overline{n}c. \quad (9)$$

Levin and Smith (1994) argue that $B(q^*, \Omega) = 0$ in equilibrium, thus affecting the seller's mechanism design. Additionally, they prove that any mechanism design that maximizes seller's payoff also induces socially optimal entry, and might involve entry fees.

4. EFFECTIVENESS OF ENTRY FEES

Main point where the two articles diverge is the role of entry fees. Menezes and Monteiro (2000) formulate the seller's maximization problem as follows:

$$\varphi(\delta) = n(n - 1) \int_{v\rho(\delta)}^{\overline{v}} (1 - F(x))\, x\, (F(x))^{n-2} f(x)\, dx + n\delta\, (1 - F(v_{\rho},(\delta))), \quad (10)$$

where δ stands for entry fee, and $v_{\rho}(\delta)$ is such that $v_{\rho}(\delta)\ (F(v_{\rho}(\delta)))^{n-1} = c + \delta$.
They prove that in an Independent Private Values (IPV) auction with endogenous bidders and no reserve price, the seller will maximize his revenue by charging an entry fee. They also indicate that this entry fee raises the seller's revenue by taking something from each bidder whose valuation is high enough. Moreover, Menezes and Monteiro (2000) suggest that first-price and second-price sealed-bid auctions can be optimal by introducing the reserve price and entry fee of zero, which is well proven in the literature by other authors.

Contrary to what Menezes and Monteiro (2000), Levin and Smith (1994) propose that the optimal entry in Independent Private Values (IPV) auctions takes place when there is no entry fee that participants have to pay. Since they proposed that reserve prices are not optimal, they prove their proposition by using equation (7), and the equation for the symmetric equilibrium (assuming that $e = 0$):

$$B(q^{*}, \Omega) = 0. \quad (11)$$

Knowing that in IPV auctions, the expected payment is on average equal to the second highest bid, and through derivation of social welfare, they prove that in such a setting, the seller should never use an entry fee nor reserve price. The main reason for such different view lies in the model settings. In Menezes and Monteiro (2000), each participant learns his value before deciding whether to bid or not to bid, after which they incur participation cost in case of entering the auction. On the other hand, in Levin and Smith (1994), participants first incur the participation cost (first step). After that, they get to decide to bid or not to bid, which in turn disables the cost to screen for low-value bidders, such as it does in Menezes and Monteiro (2000).

5. THE IMPACT OF NUMBER OF OPPONENTS ON BIDDERS

In their effort to broaden the understanding of the behaviour of the bidders in auctions, both papers deal with the impact of the number of participants on the bidder's behaviour. The setting is somewhat different than before, since bidders now know the number of opponents they face.
Menezes and Monteiro (2000) report that the bidder's behaviour in a second-price sealed-bid auction does not change regardless of the number of other participants. His dominant strategy is to always bid his true valuation. This in turn makes the cutoff point the same in case the bidder knows the number of participants and in the case he does not know.
As mentioned before, when it comes to first-price sealed-bid auctions, the dominant strategy is not to bid one's true valuation and thus is not as explicit. The strategy mainly depends on who else enters the auction.
Menezes and Monteiro (2000) derive the bidding strategy and expected revenue of bidder in equilibrium as follows:

$$\pi_i(v_i, b_i, b_{j, j \neq i}) = -c + v_i F(\hat{v})^{n-1} + \sum_{\emptyset \neq H \subset I \setminus \{i\}} (v_i - b_i^{H}(v_i))\, (F(v_i) - F(\hat{v}))^{\# H}\, F(\hat{v})^{(I \setminus \{i\} \cup Hi)}) \quad (12)$$

In the above equation, $\hat{v}$ stands for the value that sets the expected profit of participation to zero when the equilibrium number of bidders enters, and *H* for a set of participants excluding bidder *i*. They proceed with deriving the optimal bidding strategy of player *i* given *H*, only to later arrive at the expected revenue identical to the one obtained in a second-price sealed-bid auction. Thus, they conclude that first and second-bid auctions are both revenue equivalent regardless of whether the number of bidders is known or unknown.
Levin and Smith (1994) also deal with the revenue-equivalence theorem and prove that it still holds after accounting the entry fee. Similarly to the other paper, they propose that two mechanisms that are revenue equivalent with induced entry, are also equivalent with fixed *n* and *R*.

6. CONCLUSION

This paper compares two influential papers on sealed-bid auctions with entry fees: Levin and Smith (1994) and Menezes and Monteiro (2000). While much of the earlier literature assumes a fixed number of bidders, these two papers take a crucial step forward by endogenizing bidder participation - explicitly modeling how potential participants decide whether or not to enter the auction.

Levin and Smith (1994) assume that bidders must pay a fixed entry fee before learning their own valuation of the item. Once they enter, they observe how many others have also entered. Their model leads to a revenue-equivalence theorem within the Independent Private Values (IPV) framework for risk-neutral bidders. In contrast, Menezes and Monteiro (2000) propose a model in which bidders first learn their valuation and then decide whether to participate. They demonstrate that higher competition can sometimes reduce expected revenue, as entry fees serve to screen out low-valuation bidders.

Both studies analyze how the number of participants affects bidder behavior and reach a shared conclusion: for risk-neutral bidders, whether or not the seller discloses the number of participants has no impact on expected revenue. However, they diverge in their conclusions about the role of entry fees. Levin and Smith (1994) argue that entry fees are never optimal under the IPV model, whereas Menezes and Monteiro (2000) find that entry fees can be optimal—specifically when bidders know their valuations before deciding to enter. In this case, entry fees effectively screen out low-valuation bidders and boost expected revenue. But when entry decisions occur before bidders know their values, entry should not be discouraged, as the private benefits of entry align with those of the seller.

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